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10MTP11

First Semester M.Tech. Degree Examination, June 2012
Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Convert the decimal number 3.6 to corresponding binary form. **(06 Marks)**

b. Briefly explain the terms; inherent error, round-off error and truncation error. **(06 Marks)**

c. Obtain the second degree polynomial approximation to $f(x)=\sqrt{1+x}$, $x\in[0, 0.1]$ using the Taylor's series about x = 0. Use it to find f(0.05) and find the error bound. **(08 Marks)**

2 a. Solve the following system of linear equations:

$x_1 + 2x_2 - x_3 = 2,\qquad 3x_1 + 6x_2 + x_3 = 1,\qquad 3x_1 + 3x_2 + 2x_3 = 3$

by determining the inverse of the coefficient matrix. **(06 Marks)**

b. Find the solution of the following system of equations, using the Gauss-Seidel method:

$4x_1 + 11x_2 - x_3 = 33,\qquad 6x_1 + 3x_2 + 12x_3 = 35,\qquad 8x_1 - 3x_2 + 2x_3 = 20$

Carry out three iterations. **(06 Marks)**

c. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$, using the LU decomposition method.

Take $u_{11} = u_{22} = u_{33} = 1$. **(08 Marks)**

3 a. Find the largest eigenvalue in modulus and the corresponding eigenvector of the matrix using power method, $A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix}$, with initial vector $v_0 = (1, 1, 1)^T$, in three iterations. **(06 Marks)**

b. Use Given's method to find the intervals of unit length, each containing one eigenvalue of the matrix, $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. Find the largest eigenvalue correct to 1-decimal place.

(06 Marks)

c. Using the Jacobi method, find all the eigenvalues and the corresponding eigenvectors of the matrix, $A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$. **(08 Marks)**

4 a. Using the quadratic interpolation, find $y'(2.0)$ and $y''(2.0)$, given that the table of values of $y(x) = \log x$. **(06 Marks)**

x	2.0	0.2	2.6
y(x)	0.69315	0.78846	0.9555

b. The following table shows the number of employees and their daily wages in rupees. Find how many of them are getting the wage in between Rs.50 and Rs.75 per day. **(06 Marks)**

Wage (Rs.)	30-40	40-50	50-60	60-70	70-80
No. of employees	35	48	70	40	22

c. A rod is rotating in a plane. The following table shows angular displacement 'u' for various values of time 't' seconds. Calculate the angular velocity and acceleration of the rod at t = 0.6 seconds. **(08 Marks)**

t	0	0.2	0.4	0.6	0.8	1.0
u	0	0.12	0.49	1.12	2.02	3.20

5 a. Evaluate $\int_1^2 \frac{2x}{1+x^4}dx$, using Gauss-Legendre 2-point and 3-point quadrature rule. **(06 Marks)**

b. Evaluate $\int_{y=1}^{1.5}\int_{x=1}^{2} \frac{dxdy}{x+y}$, using the Simpson's rule with h = 0.5 and k = 0.25. **(06 Marks)**

c. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using the composite trapezoidal rule with 2, 3, 5 and 9 nodes and Romberg integration. **(08 Marks)**

6 a. Find the solution of the initial value problem $y' = y + x^2y$, $y(0) = 1$; taking h = 0.1 Find the y(0.1), y(0.2) using the Runge-Kutta fourth order method. **(10 Marks)**

b. Using the Adoms-Bashforth multistep method, find y(0.8) by solving $y' = -2ty^2$, $y(0) = 1$ on [0, 0.8], with h = 0.2 **(10 Marks)**

7 a. Explain the finite difference scheme (method) for solving linear second order differential equations, with boundary conditions of first kind. **(10 Marks)**

b. Solve the boundary value problem $y'' = y + x$, $y(0) = 1$, $y(1) = 0$ with h = 1/4. Use the second order finite difference method. **(10 Marks)**

8 a. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subjected to boundary conditions $u(x, 0) = \sin(\pi x)$, $0 \le x \le 1$, $u(0, t) = 0$, $u(1, t) = 0$; using Crank-Nicolson method. Carryout computations for two levels. Take h = 1/3 and k = 1/36. **(10 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square with sides x = 0, y = 0, x = 3, y = 3 and u = 0 on the boundary, take h = k = 1. **(10 Marks)**

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